

MATH 2D Review: Linear and Quadratic Curves

1. Let \mathcal{C} be the curve such that a point $P(x, y)$ is on \mathcal{C} if and only if the distance from P to the line $x = -1$ is equal to the distance from P to the point $F(1, 0)$. Find an equation for the curve \mathcal{C} , and sketch the curve. What is the name of the curve?

(Hint: Use the given information to set up an equation that x and y must satisfy)

Solution:

[distance from $P(x, y)$ to the line $x = -1$] = $x + 1$;

[distance from P to the point $F(1, 0)$] = $\sqrt{(x - 1)^2 + y^2}$

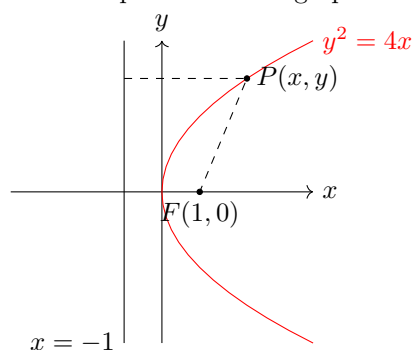
So x and y must satisfy the equation

$$x + 1 = \sqrt{(x - 1)^2 + y^2}.$$

Square both sides and reorganize, we get

$$y^2 = 4x.$$

This is a parabola. The graph is:



2. Sketch the curve $\mathcal{C} : x^2 - 4x + 4y^2 = 0$ and write down the coordinates of at least 3 points on the curve. What is the name of this curve?

Solution:

$$x^2 - 4x + 4y^2 = 0$$

$$x^2 - 4x + 4 + 4y^2 = 4$$

$$(x - 2)^2 + 4y^2 = 4$$

$$\frac{(x - 2)^2}{2^2} + y^2 = 1$$

So \mathcal{C} is an ellipse centered at $(2, 0)$. The graph of \mathcal{C} with the 3 points is:

